Statistical Modeling of Transmission Path Loss in Underwater Acoustic Networks

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Abstract—**Propagation conditions in an underwater acoustic channel are known to vary in time, causing the received signal strength to deviate from the nominal value predicted by a deterministic propagation model. To facilitate large-scale system design in such conditions (e.g. power allocation), we develop a statistical propagation model in which the transmission loss is treated as a random variable. By repetitive computation of acoustic field using ray tracing for a set of varying environmental conditions (surface height, wave activity, small displacements of transmitter and receiver around nominal locations), an ensemble of transmission losses is compiled which is then used to infer the statistical model parameters. A reasonable agreement is found with log-normal distribution whose mean is taken as the nominal transmission loss, and whose variance appears to be constant for a certain range of inter-node distances in a given deployment location. The statistical model is deemed useful for higher-level system planning, where simulation is needed to assess the performance of candidate network protocols under various resource allocation policies, i.e. to determine the transmit power and bandwidth allocation necessary to achieve a desire.**

Keywords—**Underwater acoustics, Acoustic channel model, Wireless sensor networks, Network simulation.**

I. INTRODUCTION

THE growing need for ocean observation and remote sensing has recently motivated a surge in research publications as well as several experimental efforts (e.g. [1]) in the area of underwater acoustic networks. Crucial to these developments is the understanding of propagation conditions that define the time-varying and location-sensitive acoustic environment, not only from the viewpoint of small-scale, rapid signal fluctuations that affect the performance of the physical layer techniques, but also from the viewpoint of large-scale, slow fluctuations of the received signal power that affect the performance of higher network layers. This fact has been gaining recognition in the research community, leading to an increased awareness about the need for network simulators that take into account the physics of acoustic propagation [1]-[4]. As a result, the first publicly available acoustic network simulators have emerged [2], and more are likely to come.

One of the challenges in the design of underwater acoustic networks is the allocation of power across different network nodes. This task is exacerbated by the spatial and temporal variation of the large-scale transmission loss, and the lack of statistical models that capture these apparently random phenomena.

While it is well known from field experiments that the received power varies in time around the nominal value predicted by a deterministic propagation model, little is known about the statistical nature of these variations. Literature on this topic is scarce; however, several recent references indicate that the received signal strength obeys a log-normal distribution (e.g. [5][6]). A good system design has to budget for signal strength variations in order to ensure a desired level of network performance (e.g. connectivity), and the budgeting task can be made much easier if the statistics of the underlying process are known.

In this paper, we analyze those random variations in the large-scale transmission loss that are mainly governed by the environmental factors such as surface activity (waves) for a particular network scenario. We begin by employing a prediction model based on the Bellhop ray tracing tool [7]. Such a deterministic model provides accurate results for a specific geometry of the system, but does not reflect the changes that occur as the geometry changes slightly due to either surface motion or transmitter/receiver motion. Fig.1 illustrates this situation for a point-to-point link. It shows an ensemble of transmission losses calculated by the Bellhop model for a set of varying surface conditions, each slightly different from the nominal.

While it is possible in principle to run a deterministic propagation model for a large number of different surface conditions, the underlying computational demands are high. In a large network, it is ineffective, and possibly not even feasible, to run a complex prediction model for each packet transmission. A statistical prediction model then becomes necessary.

The goal of our work is to employ an existing deterministic prediction model (DPM) such as the ray tracer [7] to generate an ensemble of channel responses corresponding to varying propagation conditions in a given network. Using the so-obtained values, we then conduct a statistical analysis to obtain the probability density function (pdf) of the large-scale transmission loss. The result is a statistical prediction model (SPM) that is easy to employ for network design and analysis. The rest of this paper is organized as follows. In Sec.II we outline a specific system example, and discuss the computational demands of deterministic propagation modeling. In Sec.III, we present the results of deterministic modeling and develop an underlying

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statistical model. In Sec.IV we discuss the implications that statistical modeling can have on network simulation, and we conclude in Sec.V.
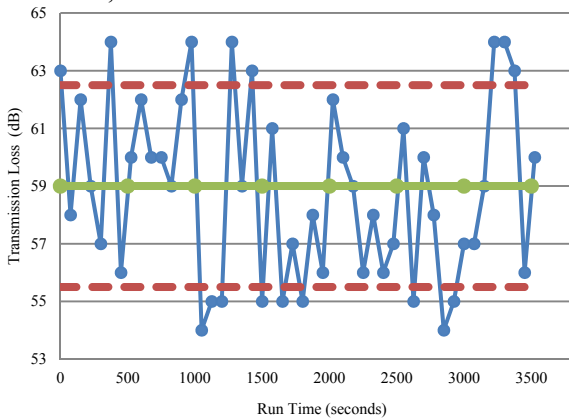

Fig. 1. An ensemble of transmission losses obtained from Bellhop model. Solid line indicates the average over the total run time. Dashed lines indicate the values of one standard deviation σ.

II. SYSTEM SET-UP

The network of interest is located in coastal waters near Valencia, Spain, at coordinates 39°48'13.14"N and 0°4'34.53"W. It consists of eight nodes arranged in a linear topology, as illustrated in Fig.2.

In general, an arbitrary network location can be considered, for which the bathymetry, floor sediment and sound speed profile can be found in the online databases [8], [9], [10]. In our example, the source is assumed to be at one end, and the rest of the nodes are placed at different distances ranging from 500m to 3,700m. The nodes are at a depth of 10 meters, while the water depth varies from 25m to 35m within the coverage area. Table I summarizes the system parameters.

We assume a fixed network topology, and vary the parameters that are related to the wave activity (wave height and wave length). The surface parameters are taken from historical and prediction values from National Geophysical Data Center databases [11], [12]. We also account for the fact that an acoustic communication signal does not consist of a single frequency, but occupies a (possibly wide) bandwidth. The overall transmission loss is thus computed over the entire frequency range, which is taken here to be 5 kHz − 15 kHz.

Each execution of the Bellhop tool [7] takes about 5 minutes on an Intel Core™ 2 Duo CPU 2.10 GHz processor running on a standard laptop computer with 3 GB of RAM memory. Considering 14 different wave heights and 14 different wave lengths, i.e. 196 different scenarios, the complete analysis lasts about 16 hours for a single source location and a single frequency within the signal bandwidth.

Each simulation run produces the acoustic field values in a 5km x 5km x 30m volume, with a resolution of $0.33m^3$. The values corresponding to selected receiving node locations are then extracted, and a statistical analysis is performed for each location.

Fig. 2. Network deployment in Valencia, Spain.

TABLE I
SYSTEM PARAMETERS

Transmission range	500 m to 3700 m (in steps of ~500 m)
Area	5000 m x 5000 m
Sediment floor	Gravel
Frequency	5-15 kHz
Month	August
Wave height	1 m to 3 m (in steps of 0.15 m)
Wave length	100 m 150 m (in steps of 3.5 m)
Water depth	25 m to 35 m

III. STATISTICAL PROPAGATION MODEL

The statistical propagation model is built by compiling the transmission loss values obtained from the deterministic model. The values of transmission loss, expressed in dB (logarithmic scale) are treated as random variables, and it is implicitly assumed that all surface conditions are equally likely.

Fig. 3 shows the histogram of the values obtained for Node 2, which is 500 m away from the source, and Fig. 4 shows the histogram obtained for Node 3, which is 1100 m away from the source. Shown also in these figures is a normal distribution with mean and variance equal to the ensemble averages of the transmission loss (solid curves).
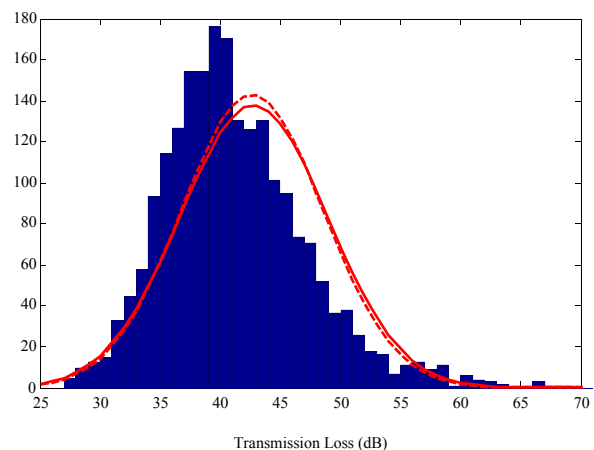

Fig. 3. Histogram of the transmission loss calculated for Node 2 using the deterministic propagation model for varying surface conditions.
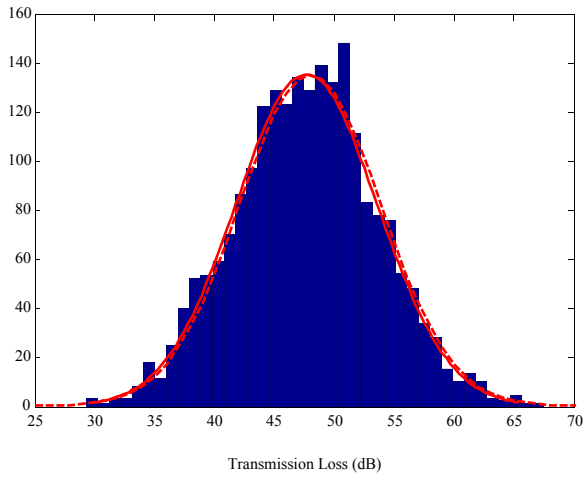
Fig. 4. Histogram of the transmission loss calculated for Node 3 using the deterministic propagation model for varying surface conditions.
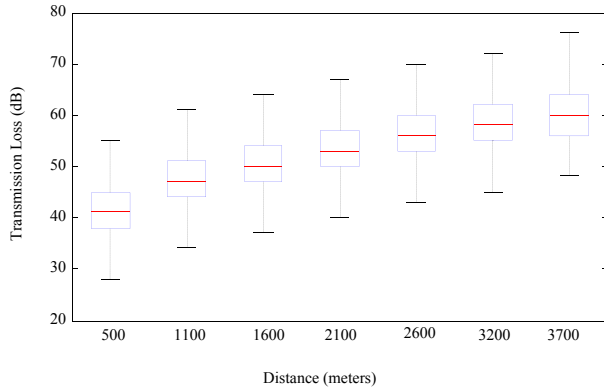

Fig. 5. Transmission loss mean value and standard deviation (boxed) versus distance.

The mean and variance obtained for different ranges from the source (different locations of the receiving node) are shown in Fig.5. We note that the mean value of transmission loss increases with distance, as dictated by the energy spreading, assumed here to be cylindrical. The variance, however, does not change much with the distance. This fact motivates us to assume the same variance for all the distances considered. Namely, we take the standard deviation to be the average of all observed values, 5.91 dB (a negligibly different result is obtained if the variances are averaged).

A statistical model is now assumed, which generates the transmission loss as a normally distributed random variable on a logarithmic scale (or equivalently, log-normally distributed on a linear scale). The mean value for this model can be taken as the ensemble average obtained from the deterministic model for a given distance, but another approach is possible as well.

Namely, we take the mean transmission loss for the statistical model to be the value obtained from a single realization of the deterministic model for nominal system geometry with no waves. The goal in doing so is to further reduce the computational demands involved in building the statistical model for a given deployment geometry.

TABLE II
TRANSMISSION LOSS PARAMETERS OBTAINED FROM THE DETERMINISTIC PROPAGATION MODEL AND STATISTICAL PROPAGATION MODEL

Node	Distance [m]	DPM		SPM	
		mean [dB]	σ [dB]	mean [dB]	σ [dB]
2	500	41.78	6.14	41.65	
3	1,100	47.82	5.91	48.01	
4	1,600	50.83	5.60	50.79	
5	2,100	53.81	6.12	53.80	5.91
6	2,600	56.52	5.74	56.32	
7	3,200	59.01	5.88	58.98	
8	3,700	60.50	5.96	60.41	

TABLE III
KULLBACK-LEIBLER DISTANCE.

Node	Distance [m]	KL distance
2	500	0.030
3	1,100	0.021
4	1,600	0.018
5	2,100	0.017
6	2,600	0.015
7	3,200	0.014
8	3,700	0.013

In Table II, we can see that nominal transmission loss (mean value listed for SPM) differs very little from the value calculated by ensemble averaging of the deterministic model's outputs (mean value listed for DPM). The pdf resulting from the statistical model is shown in Figs.3 and 4 as a dashed curve.

We note that the distributions resulting from the two models are quite similar. In order to quantitatively judge the validity of the hypothesized statistical model, we have calculated the Kullback-Leibler (KL) distance [13] between the pdf estimated from the deterministic model (histogram of Fig.3 and 4) and the Gaussian pdf used for the statistical model. This distance is zero when the two distributions are identical. In Table III, we list the KL distance for every source-destination pair considered.

IV. IMPLICATIONS FOR NETWORK PLANNING

The apparent match between the results of deterministic and statistical models motivates the use of SPM for network design and analysis via simulation. Consider, for example, network simulation over a prolonged interval of time that spans varying propagation conditions and involves transmission of a large number of data packets over multiple hops. If deterministic modeling is used, each packet transmission requires one execution of the Bellhop ray tracer, which soon becomes excessively long for a growing number of data packets (assuming 5 minutes for each Bellhop run and a single frequency, 100,000 packets would take about a year). Although the DPM offers an exact solution for the particular geometry observed at any given moment in time, its execution makes the simulation times unaffordable for benchmarking and testing of the upper layer protocols.

In contrast, a statistical model can take several hours to compute (16 hours in the example we presented) the path loss statistical model for the entire network scenario, but once computed, each realization (packet

transmission) requires only a single call to a Gaussian random number generator. Moreover, if the network topology changes slightly, or if a new node is added, the statistical model needs to be augmented only by the corresponding set of nominal transmission losses, each of which requires a single Bellhop run.

Most importantly, the statistical model can easily be used to assess transmit power allocation that will guarantee successful data packet reception with a desired level of performance (e.g. link reliability). Namely, the SPM can easily be used to calculate the transmission loss values that are not exceeded with a given probability (i.e cumulative distribution function). For example, a 90% transmission loss is that value which is not exceeded for 90% of time, i.e. in 90% of channel realizations. Fig.6 shows the 50%, 75% and 90% transmission loss for our system example. We observe a good match between the values predicted by the deterministic model, and those of the statistical model. Note that the X% values of the SPM are computed analytically, based only on the knowledge of the mean and standard deviation.

The availability of X% values is significant for determining the transmit power necessary to achieve a certain level of performance. Typically, network planning is based on the nominal ray trace, i.e. on the 50% transmission loss to which some margin may be added. If transmit power allocation is based on a different value, say 90% transmission loss instead of the nominal 50%, data packets will be more likely to reach their destinations. More power will be needed at the same time, but the overall network performance may improve. We say may improve, because a higher transmit power also implies higher levels of interference. The resulting performance trade-offs are generally hard to address analytically, and are instead assessed via simulation. A statistical propagation model that directly links the transmit power to the X% transmission loss then becomes a meaningful and useful tool for system design.
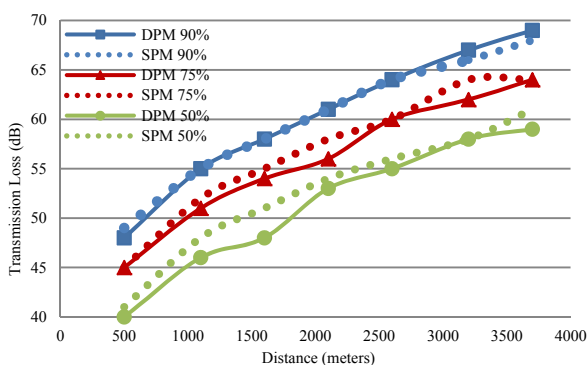

Fig. 6. Transmission loss value that is not exceeded with a given probability (50 %, 70%, 90%) is shown versus distance. The solid and dashed curves show the results obtained from the deterministic and the statistical propagation models, respectively.

V. CONCLUSIONS

Large-scale design of an underwater acoustic network requires a judicious allocation of the transmit power across different links to ensure a desired level of system performance (connectivity, throughput, reliability, etc.). Because of the inherent system complexity, simulation analyses are normally conducted to assess the performance of candidate protocols under different resource allocation policies. These analyses are often restricted to using deterministic propagation models, which, although accurate, do not reflect the randomly time-varying nature of the channel.

While it is possible in principle to examine the network performance for a large set of perturbed propagation conditions, the computational complexity involved in doing so is extremely high. To facilitate network simulation in the presence of channel fading, we investigated a statistical modeling approach. Our approach is based on establishing the nominal system parameters for a desired deployment location (water depth, sediment composition, operational frequency range) and using ray tracing to compute an ensemble of transmission losses for typical inter-node distances. An ensemble is generated by considering a set of perturbed surface conditions, defined by varying wave activity (height, period). The so-obtained ensemble is then used to determine the statistical parameters of a hypothesized log-normal distribution of the transmission loss. For a representative example of a small network operating in a 5 km x 5 km area with inter-node distances ranging between 500 m and 3.5 km, it was found that the mean can be well approximated by the value obtained using nominal system parameters, while the variance can be modeled as distance-independent. Models that are more elaborated and more accurate than the log-normal one can also be developed using this approach.

This kind of statistical modeling allows computationally-efficient inclusion of fading effects into a network simulator. Namely, to assess the average system performance, network operation has to be simulated over a large set of channel realizations (e.g. varying surface conditions). Whereas repeated computation of the ray trace for different hops that each of the data packets traverses in a given network may be computationally prohibitive, statistical modeling requires only a single call to the Gaussian random generator for each packet transmission. The overall simulation time is thus considerably reduced, allowing a system designer to freely experiment with varying protocols and resource allocation strategies in an efficient manner. The ultimate goal of such experiments is to choose the best upper-layer protocol suite and to relate the necessary system resources (power, bandwidth) to the propagation conditions, i.e. to the *statistical* parameters of the transmission loss (e.g. X% value), which can in turn be easily generated using the proposed method of statistical modeling

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