Adaptive dead zone estimation for wavelet based encoder

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Abstract— **Rate/Distortion performance is enhanced with perceptual coding and quantization techniques, as luminance and contrast masking, or with the use of frequency weighting matrices obtained from the Contrast Sensitivity Function (CSF). These techniques reduce the rate of images or videos while preserving the perceived visual quality. Encoders that use some of these techniques should be compared with perceptual R/D curves i.e., where the distortion metric is an objective quality assessment metric. In this paper we introduce a method used to obtain image adaptive dead zone size estimators that provides a dead zone value that maximizes the perceptual R/D performance. We use a wavelet based encoder that is perceptually enhanced with a frequency weighting matrix directly obtained from a model of the CSF to test our proposal. Results show that when using the best performing estimator, at the same perceptual quality, mean rate savings up to 9.98% can be achieved.**

Keywords— **Perceptual coding, perceptual quantization, R/D performance, optimum dead zone quantizer.**

I. Introduction

THE S-LTW encoder was presented in [1]. It is a wavelet based encoder with lower resource demanding than other encoders in the literature. The basic idea of this encoder is very simple: after computing a dyadic wavelet transform of an image, the wavelet coefficients are first quantized and then encoded with an arithmetic coding. In S-LTW, the quantization process is performed by means of two strategies: one coarser and another finer. The finer one consists on applying a scalar uniform quantization (Q) to wavelet coefficients. The coarser one is based on removing the least significant bit planes (rplanes) from wavelet coefficients.

Although having a good R/D performance this encoder was not enhanced with any perceptual coding technique. Perceptual coding techniques are those that include some of the properties of the Human Visual System (HVS) in their design, using the knowledge of how the HVS process natural scenes along the visual pathway, to encode images and video sequences in a perceptually inspired way.

Using these techniques, the so named ”peceptually enhanced” encoders, are able to reduce the bit rate needed to encode natural images by discarding irrelevant information to our HVS that is present in the transformed coefficients. The most commonly used perceptual coding techniques are the inclusion of the CSF, luminance masking and/or contrast masking into the encoder, what can be achieved in different ways as explained in [2]. The base sensitivity thresholds, obtained via perceptual subjective tests or derived from a CSF model, are typically used to fully quantize, i.e., remove, the transformed coefficients that are below threshold, and that are supposed to correspond to perceptually redundant information.

Therefore a motion version of the S-LTW encoder, was enhanced in [3], with the inclusion of the CSF in the quantization stage of the encoder . The reader is refered to [3] for details of how the inclusion of the CSF was made and which model of CSF was used. The proposed encoder achieved up to 19.42% of bit rate saving for HD resolution sequences with respect the X.264 video encoder at the same perceptual quality. As the new proposed encoder used perceptual coding techniques the results should be compared using a perceptual objective Quality Assessment Metric (QAM) ([4], [5]). Results were measured by means of the VIF quality assessment metric that was the one that best performance obtained in previous metric comparison tests [6].

The performance of encoders using dead zone quantizers can be further increased by tuning the dead zone size. In [7], Ström made an experiment with one image to determine how large the dead zone should be in that image for optimal performance, and how much quality could be gained when the Uniform Scalar Quantizer (USQ) is substituted with a Uniform Scalar Dead Zone Quantizer (USDZQ) in a DWT encoder. Their study was done also in terms of R/D performance with the PSNR as quality metric. As mentioned, they used only one image resulting an optimal dead zone size of 1.9Δ , which finally provides a quality increase of 0.5 dBs for that image.

As in the Ström experiment, we will also use a DWT based encoder, but in this case our PETW proposal [8] that implements a Uniform Variable Dead Zone Quantizer (UVDZQ). We use the VIF metric because the optimum dead zone should be calculated taking into account the perceptual elevation produced by the Perceptual Weighting Matrix (PWM) that sets the CSF into the quantization stage in the encoder. With a maximization algorithm an optimum dead zone size could be reached for each individual image at a fixed Qstep, but from a practical use this is a very high time consuming task. More over, the R/D curve is obtained varying the Qstep and this process should be repeated for each Qstep.

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In this paper we present how we obtain three dead zone size estimators that produces a near optimum dead zone size, for all the bit rate range used in the R/D curves, being in addition image adaptive and with very low computational cost. The dead zone estimator uses the ξ parameter of the UVDZQ that produces the best perceptual R/D curve.

The rest of the paper is structured as follows. In Section II, a short introduction to the PETW encoder is presented, in Section III a brief review of the related quantization schemes is presented. The dead zone size estimators are presented in Section IV, while some results are presented in Section V. Finally Section VI concludes the paper.

II. Perceptually Enhanced Tree Wavelet Encoder - PETW

The LTW and the S-LTW encoders employ a quantization mechanism based on two parameters [9], one finer (Q) and another coarser ($rplanes$). Thus, the quantized image is the result of jointly applying two quantization methods. The first parameter performs a scalar quantization with a step-size of $2Q$, and the second one consists on removing the $rplanes$ least significant bits of all coefficients, being a simple bit-plane quantization process. The two quantization stages of the S-LTW act jointly as a Uniform Dead Zone Quantizer (UDZQ), and therefore the use of both quantization processes may seem a bit strange, but it reveals more natural when the LTW is studied in depth, as some coding optimizations can be included as a result [9].

The new encoder proposal called Perceptually Enhanced Tree Wavelet (PETW) [8] is based also on the S-LTW encoder, that besides having the perceptual weighting stage (by the use of the PWM), it has a new quantization strategy based on a UVDZQ, so reducing to one the parameters needed by the encoder to control the quantization stage. Setting in the UVDZQ the equivalent dead zone size that the S-LTW uses, allow us to use only the step-size parameter Q to control the amount of quantization, providing the same results as when no perceptual enhancement is applied. The quantizer change also enables us to obtain encoded images with higher rates than with the S-LTW encoder, as we do not have the restriction imposed by the coarse quantizer that is always applied with a minimum value of $rplanes = 2$. Reaching higher rate ranges is appropriate when working in the sub-threshold or visually lossless area, where distortions are supposed not to be detected in static images by humans.

The motivation for changing the S-LTW quantization stage is based on the work of [10], where authors made several performance comparisons between a USQ, a USDZQ, and a Universal Trellis Coded Quantizer (UTCQ), using the same step size, and applied to DWT and DCT transformed coefficients. Their performance comparisons show that the UTCQ can quantize data more precisely and provide better PSNR results than the other two quantizers when using the same step size. But, when they are combined with zero or higher order entropy coders, the dead zone quantizer (the USDZQ) is the best instead. In these comparisons, authors show that if the dead zone is designed carefully, the USDZQ can effectively reduce the output hits of the entropy coder, and although it reduces quantization precision by discarding some data around zero, the obtained rate reduction is worthwhile. Moreover, the USDZQ is only a USQ with a dead zone, and its computational complexity is lower than the UTCQ.

Our studies are oriented to optimize the R/D behavior in terms of the VIF QAM, and hence to determine which is the influence of the dead zone size over the perceptual quality, and not over the PSNR as in previous studies. The variable dead zone schema that we use in the PETW encoder is the one proposed in the JPEG2000 encoder [11], [12].

III. Quantization

In this section we shortly review the S-LTW 2-stage quantizer in contrast to the UVDZQ in order to determine the parameters that equals both quantization strategies from a PSNR point of view. In the next section (Section IV), we will see how the dead zone variability allowed by the UVDZQ can be used to enhance the visual perception of images by setting the optimal dead zone size in each case.

In order to change the S-LTW quantizers with a UVDZQ, the first step is to find the correspondence with UVDZQ. To do so, we use the next formulation, (see [9] for detailed formulation of the 2-stage quantization of the S-LTW). The overall step size Δ applied to S-LTW can be viewed as the multiplication of two deltas, Δ_1 corresponding to the finer quantizer and Δ_2 corresponding to the coarser one, as shown in Equation 1.

$$\begin{aligned} \Delta &= \Delta_1 \cdot \Delta_2 \\ \Delta_1 &= 2Q \\ \Delta_2 &= 2^{rplanes} \end{aligned} \qquad (1)$$

In order to replace these two quantizers with a UVDZQ, we must know the relationship between the dead zone size, and the overall Δ applied in S-LTW, i.e., to obtain the equivalent dead zone size in the UVDZQ that is used in S-LTW.

Equation 2 is the forward quantizer expression for a UVDZQ that sets the value c_Q of the quantized coefficient [12]. The parameter ξ, so that $\xi < 1$, determines the size of the dead zone in such a quantizer. Depending on the value of this parameter, the dead zone size is set as follows:

- $\xi < 0$ increases the dead zone size above the size of 2Δ
- $\xi = 0$ produces a dead zone with double the size as the quantization step, i.e., 2Δ and then the

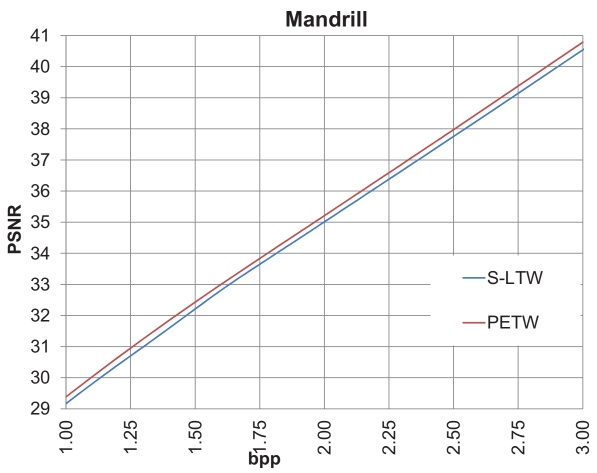

Fig. 1. Equivalence of the R/D behavior between S-LTW joint quantization and the PETW dead zone quantization for Mandrill.

upper bound of the positive part of the dead zone is Δ

- $0 < \xi < 1$ reduces the dead zone so that its size is lower than 2Δ. A typical value is $\xi = 0.500$, which produces a dead zone size of Δ

$$c_Q = \begin{cases} sign(c)\left\lfloor \frac{|c|+\xi\Delta}{\Delta} + \rho \right\rfloor & if\ \frac{|c|}{\Delta} + \xi + \rho\ > 0 \\ 0 & Otherwise \end{cases} \quad (2)$$

$$c_R = sign(c)\,(|c_Q| - \xi + \delta - \rho)\,\Delta \quad (3)$$

The DZ size of S-LTW related with the overall step size Δ is $DZ = 1.25\Delta$, so we must set the correct parameters in the UVDZQ formulation, so that its dead zone size is also $DZ = 1.25\Delta$. Then, we can check if the results obtained with the PETW encoder without perceptual enhancements but with the UVDZQ are the same as those obtained with S-LTW. Finally, we will proceed with the performance analysis.

The ρ parameter in Equation 2 determines if we will finally use a truncation operation or a rounding one in the quantizer, see Equation 4, and also the $0 \leq \delta < 1$ parameter sets the recovering point inside the quantization interval.

$$\rho = \begin{Bmatrix} 0 & for\ truncating \\ 0.5 & for\ rounding \end{Bmatrix} \quad (4)$$

So, if for S-LTW we have dead zone size of 1.25Δ **??**, we must use a ξ value so that $0 < \xi < 1$. To use a UVDZQ that equals the behavior of both quantizers of the S-LTW acting together we must fix $\xi = 0.375$ and $\rho = 0$. With Equation 3, we can finally obtain the reconstructed coefficient c_R.

In figure 1 we can see that the PSNR R/D behavior of the equivalent PETW, with a dead zone size of $DZ = 1.25\Delta$ obtained with a $\xi = 0.375$, is almost the same as the one obtained with the original joint quantization of S-LTW. For some images with high frequency content, as those shown in Figure 1, the obtained PSNR is slightly better with the new quantization schema of PETW. This is because in S-LTW, the rate control is enabled and the *rplanes* parameter changes depending on the desired rate. This is an expected result as stated by the S-LTW authors because fixing the $rplanes = 2$ and then increasing the finer quantizer up to the desired rate produces slightly better results.

IV. Dead Zone Estimation

Our objective in this section is focused into the impact of the dead zone size on the R/D coding performance. So, we will analyze how different dead zone sizes affect to the VIF R/D performance, and then we will propose a way to estimate the dead zone size that maximizes the VIF R/D performance and therefore, the perceptual quality.

In order to determine the optimum ξ value for a specific image, we performed an experiment to obtain this ξ parameter from a R/D point of view. We choose five step sizes, i.e., five values for the Q parameter of the PETW that produce five rates evenly spaced in the bit rate range. For each Q value we encode and decode the image and obtain the real VIF value at the corresponding rate. With the real VIF/rate values we use Equation 5 to estimate the VIF R/D curve for those points. We propose Equation 5 as one approximation to the R/D curve when the employed distortion metric is VIF, see [8] for more details. Using this approximation, we produce 101 estimated curves for each image, one curve for each ξ value in the range $-0.500 <= \xi <= 1$ chosen in increments of 0.010 units. We call these curves *Xi Curves*. Doing so, the only parameter that changes in the PETW encoder is the ξ one, as the step sizes for each of the curves are the same.

$$VIF(r) = \frac{p_1 \cdot r^2 + p_2 \cdot r + p3}{r + q_1} \quad (5)$$

Then, for each of the *Xi Curves*, we obtain the bit rate gain or loss using the same method as in Bjontegaard [13], [14]. We compare the gain or loss of each curve with the one obtained with the reference curve. The reference curve is the one obtained with $\xi = 0.375$ that equals the 1.25Δ dead zone size of the S-LTW. We used the whole image Kodak set to obtain the optimum ξ value for each image, we call it *best xi value*, i.e., the value that maximizes the bit rate gain with respect to the reference R/D curve in the VIF range from 0.30 to 1.0 VIF units.

Table I shows the best xi values for the Kodak set images. The objective is not to perform these calculations for each image but to find an adaptive method or equation. First, we search for one value that could be calculated on the fly or used as a well-working global value.

One way to avoid the task of calculating the best xi for every image is to obtain a unique value that is sub-optimum for the image. One candidate value

TABLE I
Best xi values for the Kodak set images.

Image	Best Xi	Median Err.	Mean Err.
1	0.340	0.220	0.263
2	-0.360	0.480	0.437
3	-0.190	0.310	0.267
4	-0.130	0.250	0.207
5	0.320	0.200	0.243
6	0.270	0.150	0.193
7	0.120	0.000	0.043
8	0.250	0.130	0.173
9	-0.030	0.150	0.107
10	-0.070	0.190	0.147
11	0.160	0.040	0.083
12	-0.220	0.340	0.297
13	0.410	0.290	0.333
14	0.220	0.100	0.143
15	-0.100	0.220	0.177
16	0.050	0.070	0.027
17	0.080	0.040	0.003
18	0.260	0.140	0.183
19	0.170	0.050	0.093
20	0.080	0.040	0.003
21	0.270	0.150	0.193
22	0.130	0.010	0.053
23	-0.250	0.370	0.327
	Avg.	0.171	0.174

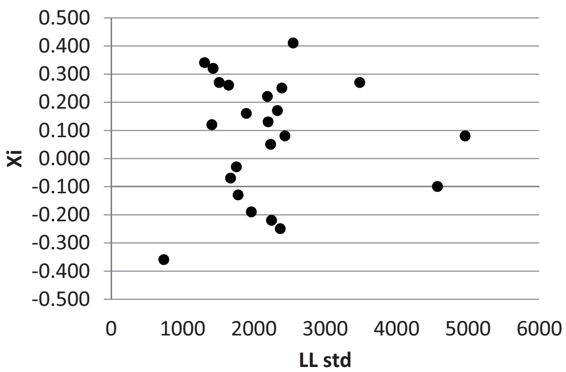

Fig. 2. Dispersion plot for the *best xi* vs. LL std for images in the Kodak set.

to use is the mean or median ξ value of column *best xi* in Table I. The mean value is 0.077 and the median value is 0.120. In Table I, the *Median Err.* and *Mean Err.* columns show the estimation error between these values and the optimum xi value from the *best xi* column. The last row shows the mean error for each of these estimated values. Although for some images these estimated xi values produce practically the same VIF R/D curve than the one obtained with the best xi value, none of them is a good approximation because for other images the R/D curve is below the reference one.

So the objective is to find another estimated xi value that is able to minimize this averaged error. We then searched for a correlation between the best xi values and some statistical value or metric obtained directly from the image, or from the wavelet coefficients before the quantization is performed, as the ξ parameter must be known at this point.

A first option is to use the Standard Deviation (SD) of the wavelet coefficients, but as shown in

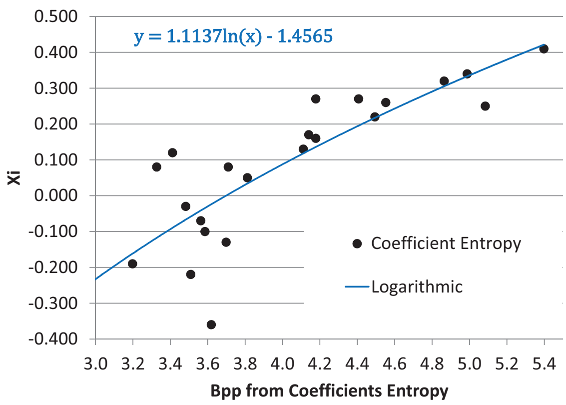

Fig. 3. Scatter plot for the Best Xis vs. E_{bpp} obtained with the *Coefficient Entropy* estimator for images in the Kodak set. Logarithmic fitting equation is also shown.

Figure 2, where the SD of the LL subband is used, there is no appreciable correlation between the best xi values from Table I and the SD for each image, shown on the horizontal axis. So we proceeded to search for some entropy measures, which we call *estimators*, that are able to estimate the bpp used for each image, producing an estimation of the bpp value, E_{bpp}. These estimators were implemented in the PETW before the quantization stage, and therefore before the encoding stage. We implemented three estimators:

- *Coefficient Entropy*: This is the zero order entropy obtained directly from the wavelet coefficients after transform. This is a generic measure that does not depend on the encoder.
- *Symbols Entropy*: This is the zero order entropy of the PETW symbol map used in the encoding process. This measure depends strictly on the PETW encoder as the symbols will be used by the encoding algorithm.
- *PETW Bpp*: This is an entropy estimation that uses the E_{bpp} produced by *Symbols Entropy* plus the real amount of bpp used for the raw bits of each of the coefficients. In order to determine the real bits needed for each coefficient, a dead zone size and a step size must be fixed. We use a dead zone size equivalent to the one used by the rate control stage in the S-LTW, which uses a $rplanes = 2$ with no further quantization. This estimator is also dependent on the PETW.

Once we have the E_{bpp} from each estimator, we use a scatter plot to see if there is some correlation between the E_{bpp} and the optimum xi for each image in the Kodak set. In figures 3 to 5, we see these scatter plots, where a correlation is shown.

Figures 3 and 4 also show the best fitting equation (logarithmic in both cases), that is used to estimate the best xi values for a desired bit rate. In Figure 5, a polynomial fitting is shown instead.

Table II shows the results for the *Coefficient Entropy (C.)* and the *Symbols Entropy (S.)* and the

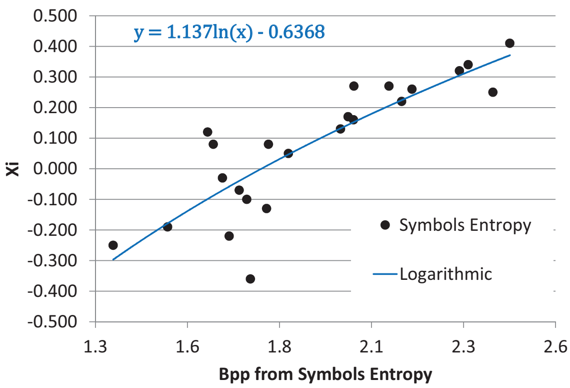

Fig. 4. Scatter plot for the Best Xis vs. E_{bpp} obtained with the *Symbols Entropy* estimator for images in the Kodak set. Logarithmic fitting equation is also shown.

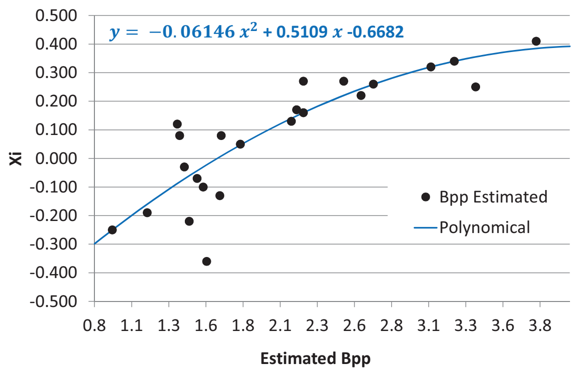

Fig. 5. Scatter plot for the Best Xis vs. E_{bpp} obtained with the *PETW Bpp* estimator for images in the Kodak set. Polynomial fitting equation is also shown.

PETW Bpp (P.) estimators. Columns are the Kodak set image number, the previously obtained best ξ and the errors with respect to the optimum ξ using the fitting equation for each estimator. In the last row, the average error is also shown for each fitting equation.

The worst results are obtained with the *Coefficient Entropy*, which is the only encoder-independent estimator. However, this is a much better estimator than using the Mean or the Median of the optimum xis. Therefore, this estimator could be used in any wavelet based encoder that uses a dead zone quantizer, like for example JPEG2000, although some adaptations and more experiments must be done.

In the case of the PETW-dependent estimators, the best results are obtained with the *PETW Bpp*, which is also the better of the three estimators that we have implemented. It obtains an average error of 0.069ξ. The polynomial fitting equation used in *PETW Bpp* is also shown in Equation 6, where $E\xi$ stands for estimated Xi, and E_{bpp} is the estimated bpp obtained with it.

$$E\xi = -0.06146E_{bpp}^2 + 0.5109E_{bpp} - 0.6682 \qquad (6)$$

TABLE II

IMAGE AND AVERAGE ERROR FOR THE FITTING EQUATION.

Image	Best Xi	C. Entropy Err.	S. Entropy Err.	P. Bpp Err.
01	0.340	0.007	0.024	0.000
02	-0.360	0.336	0.340	0.338
03	-0.190	0.028	0.011	0.030
04	-0.130	0.130	0.139	0.136
05	0.320	0.015	0.015	0.000
06	0.270	0.075	0.065	0.049
07	0.120	0.210	0.219	0.208
08	0.250	0.104	0.099	0.105
09	-0.030	0.037	0.041	0.042
10	-0.070	0.029	0.030	0.027
11	0.160	0.024	0.008	0.000
12	-0.220	0.162	0.162	0.159
13	0.410	0.011	0.039	0.025
14	0.220	0.003	0.004	0.024
15	-0.100	0.065	0.074	0.070
16	0.050	0.016	0.003	0.003
17	0.080	0.076	0.068	0.072
18	0.260	0.029	0.021	0.001
19	0.170	0.044	0.026	0.021
20	0.080	0.198	0.168	0.162
21	0.270	0.134	0.117	0.110
22	0.130	0.012	0.002	0.010
23	-0.250	0.013	0.047	0.000
	Avg.Err.	0.076	0.075	0.069

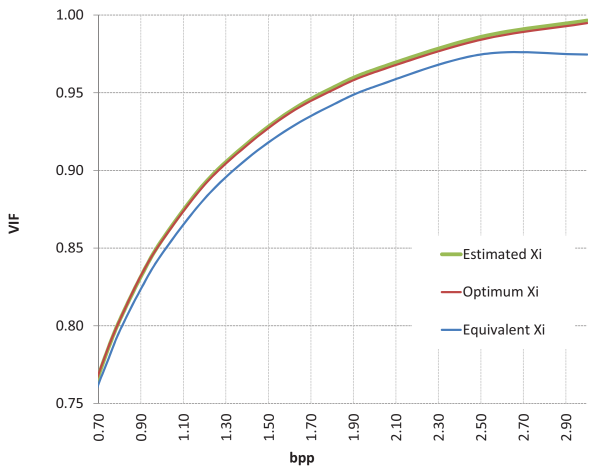

Fig. 6. Lena: R/D curve comparison between the Optimum Xi, Estimated Xi, and Equivalent Xi values.

V. RESULTS

Once we have an equation to estimate the best ξ for a specific image, we will test it with other well-known 512x512 size images, Lena, Barbara and Zelda.

As an example of the estimation performance oft the *PETW Bpp* via Equation 6, Figure 6 shows how the Estimated Xi R/D curve has the same behavior than the R/D curve obtained for the Optimum Xi value whereas Equivalent Xi value obtains a lower R/D curve.

A segmentation of the VIF scale is performed in order to be able to measure the average bit rate savings for each quality segment using the methods proposed in some refinements of the Bjotegaard model [14]. The segmentation is defined for the next quality ranges:

- Visually Lossless (VL): $VIF >= 0.83$

TABLE III
ADDITIONAL % OF BIT RATE GAIN/LOSS DUE TO THE USE OF THE *PETW Bpp* ESTIMATOR FOR THE VL AND E QUALITY RANGES.

Images	Visually Lossless		Excellent	
	% Add.	% Tot.	% Add.	% Tot.
Lena	**3.80%**	18.08%	**1.89%**	14.59%
Barbara	1.18%	12.35%	0.84%	14.32%
Zelda	**6.50%**	23.50%	**3.54%**	16.70%

TABLE IV
ADDITIONAL % OF BIT RATE GAINS/LOSSESS DUE TO THE USE OF THE *PETW Bpp* ESTIMATOR FOR THE G AND A QUALITY RANGES.

Images	Good		All	
	% Add.	% Tot.	% Add.	% Tot.
Lena	-0.95%	7.74%	0.98%	12.46%
Barbara	0.37%	18.42%	0.68%	15.81%
Zelda	-0.23%	4.97%	2.40%	13.36%

- Excellent (E): $0.60 <= VIF <= 0.83$
- Good (G): $0.30 <= VIF < 0.60$

In Tables III and IV, we show in the column labeled as *%Add.* the percentage of additional gain or loss that could be obtained using the estimator in the PETW with respect the PETW without the estimator. In the column labeled as *%Tot.* the percentage of bit rate saving that is obtained with respect to the S-LTW encoder, i.e., values in the *%Tot.* are the bit rate savings when for PETW, the proposed dead zone estimator and the PWM are jointly used.

Table III shows the values corresponding to the *Visually Lossless* and *Excellent* quality ranges, whereas Table IV shows the values corresponding to the *Good* and *All* quality ranges, covering with the All range the union of the three considered quality ranges.

VI. CONCLUSIONS

In this paper we present how in a wavelet based encoder, a change of an Uniform Scalar Quantization stage with a Uniform Variable Dead Zone Quantizer, can increase the perceptual performance when choosing the right dead zone size. Here, the original encoder was the S-LTW encoder where the 2-stage quantization stage has been changed with an equivalent variable dead zone quantizer. In the new encoder, the PETW, the dead zone size was initially set to produce the same quantization as the original uniform quantizer. Then, by running a comprehensive experiment an optimum dead zone size could be found for each individual image so that the perceptual R/D performance is maximized. In order to avoid the need to search this optimum dead zone size for each individual image, three dead zone size estimators are presented. The use of the best performing one into the encoder, includes image adaptivity when determining the dead zone size.

Results confirm the importance of using an optimum dead zone size for each image to obtain a better quality of the reconstructed image. The image adaptive dead zone size estimator is developed in order to obtain the best R/D performance when the distortion metric is the VIF metric. The methods used in this proposal can be extrapolated for use any other distortion metric instead. Several estimators were tested and the best performing one is a PETW encoder dependent. One of the proposed dead zone size estimators is, however encoder independent, so with some adaptations it could be used in other wavelet and DCT-based encoders.

The use of the image adaptive dead zone size estimator in the PETW produces additional bit rate savings, and, depending on the image, up to 9.98%, 6.48%, 0.97%, and 4.73% in the *Visually Lossless*, *Excellent*, *Good*, and *All* quality ranges, respectively.

The PETW with the image adaptive dead zone estimator, is very competitive in terms of perceptual quality, measured with the VIF QAM, being able to obtain important bit rate savings regardless of the bit rate, when compared with S-LTW.

These savings highlight the benefits that an appropriate selection of the dead zone size introduces in the perceptual performance of the decoded images.

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